EKF based slope angle estimation on TWIP

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***Abstract*—In this paper, we propose a model-based road slope estimator for a Two Wheeled Inverted Pendulum.**

I. Introduction

Over the last few years, the number of industrial applications that could benefit from the use of mobile robots has been steadily increasing. Last mile delivery is good example of a field in which fast, compact and agile autonomous vehicles are regarded as a key enabler for effectively developing a service of autonomous urban delivery. Among robots designed for this specific application, a particularly interesting structural choice is that of Two Wheeled Inverted Pendulums (TWIP), which are self-balancing, two-wheeled robots capable of turn-on-the-spot maneuvers. TWIPs enhanced agility makes them the perfect choice for use cases in which small turning radius maneuvers are required, while the absence of a steering mechanisms allows for a very high ratio between available cargo loading volume and overall robot dimensions. However, their intrinsic instability presents some challenges from the control point of view, especially when they are employed on challenging terrains such as urban sidewalks. The presence of irregular pavement, potholes, small steps and slope variations makes controlling a TWIP in a urban environment a non trivial problem [3]. This work addresses the design of a model based observer for the estimation of the road slope to be used on TWIPs. The availability of an estimate of the road inclination brings several advantages to the TWIP control process, both in terms of improved reference tracking performance and of reduced risk of crashes due to over-tilting. Nonetheless, in the vast TWIP control literature [1] non-zero road slope angle is commonly treated as an unknown disturbance to be rejected. The main reason for this is that slope angle estimation on TWIPs is a non-trivial problem. The complexity of estimating the road angle on TWIP stems from the fact that the pitching motion amplitude in a TWIP is substantially bigger than the one occurring on a standard four-wheels vehicle, and therefore cannot be considered negligible in the estimation process. This makes most of the techniques for road slope estimation developed in the automotive fields [4] [5] [2] not applicable to this case. We decided to tackle the estimation problem with a model-based approach which considers the specific dynamics of a TWIP in presence of non-zero road inclination. In particular, our work deals with developing an Extended Kalman Filter (EKF) based strategy that also accounts for the presence of dissipative effects such as hub friction and rolling resistance.

Fig. 1. YAPE: The prototype used in the experimentation

A. Experimental Setup

This research was conducted on YAPE (Fig. 1), a TWIP designed for autonomous parcel delivery in urban environment [3]. From a structural point of view YAPE is composed of a steel chassis and two parallel driving wheels, and it is equipped with both exteroceptive and proprioceptive sensors. Among the latter, our estimation algorithm leverages two types of sensors: a 6-axis (3 accelerometers and 3 gyrometers), chassis-fixed Inertial Measurement Unit (IMU) mounted at the axle center, and two rotational encoders, one per wheel, measuring the relative rotation between the chassis and each wheel. Mainly due to their relatively low cost, these kinds of sensors are very commonly found on the vast majority of mobile robots, which supports the applicability of our work to prototypes other than our own.

B. Observer structure

Fig. 2 depicts the block diagram of the observer-controller structure implemented on YAPE. We opted for a decoupled approach which leverages a series of two separate observers: the first is in charge of estimating the vehicle dynamical state ($\hat{y} = [\hat{s}, \hat{\theta}, \hat{\dot{\theta}}]$), while the second computes the actual estimate

$$
\begin{cases}
\left(\frac{2J_w}{R^2}+2m_w+m_c\right)\ddot{s}-m_cL\theta^2\sin(\alpha+\gamma+\theta)+m_cL\ddot{\theta}\cos(\alpha+\gamma+\theta)-\frac{\tau_r}{R}-\frac{\tau_l}{R}+\frac{\tau_{hinge}(\dot{s})}{R}+\frac{\tau_{roll}}{R}+(2m_w+m_c)g\sin(\alpha)=0 \\
J_c\ddot{\theta}+\tau_l+\tau_r-\tau_{hinge}(\dot{s})+m_c\ddot{s}\cos(\alpha+\gamma+\theta)L+m_cL^2\ddot{\theta}-m_cgL\sin(\gamma+\theta)=0
\end{cases}
\tag{1}
$$

for the slope angle α. This makes our slope observer easy to be embedded into a preexisting TWIP control structure, in which both u and $\hat{y}$ are supposed to be already available in order to achieve a closed-loop stabilization of the chassis.

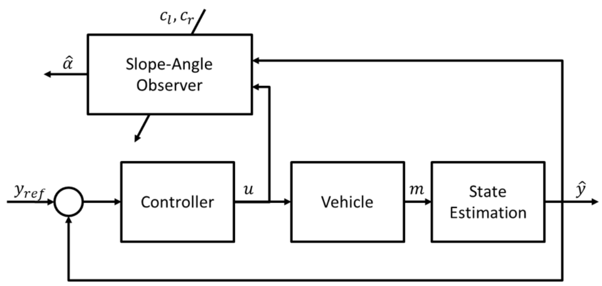

Fig. 2. Block diagram of the slope angle ($\hat{\alpha}$) observer. The vector $u = [\tau_r, \tau_l]$ contains the torque measurements for each wheel, $\hat{y} = [\hat{s}, \hat{\theta}, \hat{\dot{\theta}}]$ the estimated dynamical state and m represents the vector of sensor measurements.

C. Dynamical model

Our observer is based on a nonlinear model of the second order (1).

Notice how (1) explicitly includes two torque components that model the main dissipation effects acting on the vehicle, namely the wheel hub friction τ_{hinge} and the rolling resistance τ_{roll}. In particular, the first is modeled as an internal torque generated in the wheel hubs acting both on the wheels and on the chassis, while the second introduces a torque component acting on the wheels only. Equation (1) shows how the effect of these dissipation phenomena on the vehicle dynamics is matched with that of the slope angle; therefore the quality of the final estimate is strictly linked to how well these effects are modeled.

Hinge friction is computed using a piece-wise linear model depending on the relative rotational speed between the wheels and the chassis:

$$
\tau_{hinge}(\dot{s},\dot{\theta}) = \begin{cases} (a_f(\frac{\dot{s}}{R}-\dot{\theta})+b_f) & \text{if } \frac{\dot{s}}{R}-\dot{\theta} >= 0 \\ (a_b(\frac{\dot{s}}{R}-\dot{\theta})+b_b) & \text{if } \frac{\dot{s}}{R}-\dot{\theta} < 0 \end{cases} . \tag{2}
$$

Rolling resistance on the other hand is modeled as a constant term always acting against the direction of motion:

$$
\tau_{roll}(\dot{s}) = \text{sign}(\dot{s})(c_l + c_r) \tag{3}
$$

The parameters in (2) were experimentally identified offline and considered to be constant thereafter (gears wear was deemed to be negligible within the scope of this work), whereas c_l and c_r in (3) were estimated online with an ad-hoc procedure.

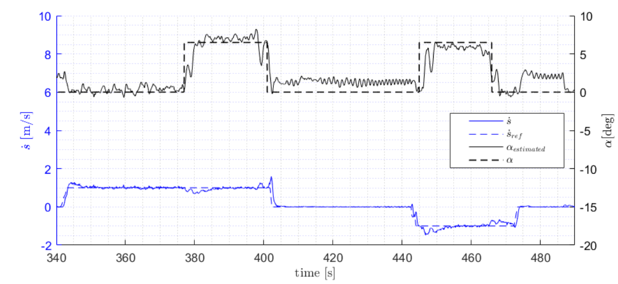

Fig. 3. Experimental results obtained with our slope angle observer. The estimated value for α is reported in black (solid line). The corresponding ground truth value was obtained offline by measuring the average slope with a digital level. Vehicle estimated speed (solid blue line) and the corresponding reference value (dotted blue line) are also reported.

D. Experimental results

Fig.3 reports the results obtained by running the prototype on a sloped road with measured inclination of 6°. The experiment was conducted by driving the vehicle uphill in forward direction, reaching the top of the slope and then driving back downhill in reverse. The ground truth profile for the slope angle was obtained by combining the average measured value with a recorded manual trigger marking the beginning and the end of the slope. The estimate produced by our observer presents very limited steady-state error and a satisfactory dynamical behavior, with a rising time under 5 seconds.

Our work further analyses the quality of these results by investigating the usability of the estimate to improve the reference tracking performance of the underlying vehicle control loop.

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