Vibrational model of an articulated robot arm

Matteo Bottin
Department of Industrial Engineering
University of Padova
Padova, Italy
matteo.bottin@unipd.it

Silvio Cocuzza
Department of Industrial Engineering
University of Padova
Padova, Italy
silvio.cocuzza@unipd.it

Alberto Doria
Department of Industrial Engineering
University of Padova
Padova, Italy
alberto.doria@unipd.it

Giulio Rosati
Department of Industrial Engineering
University of Padova
Padova, Italy
giulio.rosati@unipd.it

***Abstract*—The vibrational behavior of a robot arm plays an important role in some industrial applications, such as machining. In this paper, the variation of the inertia terms of a robot arm is investigated with the creation of a mathematical model. Validation tests are provided.**

***Index Terms*—robot, inertia, modes of vibration.**

I. Introduction

In intensive industrial applications, such as machining, very few industrial robots are used, mainly due to their low stiffness [3]. In fact, robot compliance may generate chatter vibrations, with a consequent reduction in surface finishing.

To overcome this problem few solutions have been proposed [1], [2], but most of them rely on the adoption of additional hardware to increase the manipulator stiffness.

However, the intrinsic properties of the robot allow to modify the inertial data of the structure: if an industrial robot (Figure 1) is fully extended on the horizontal plane, the moment of inertia about joint 1 reaches the maximum value; conversely, if the robot is on a position close to robot's base, the inertia with respect to joint 1 axis would be much lower.

In this sense, this paper would like to investigate how the configuration, and as a result the components of the inertia matrix, influences the vibrational properties of a 6-axis robot.

II. Dynamic model

Let's consider a configuration described by a set of joint variables $\boldsymbol{q}_0$. In a study of vibrations for the development of a dynamic model, the joint variables, velocities and accelerations around this configuration are defined as:

$$\boldsymbol{q} = \boldsymbol{q}_0 + \Delta\boldsymbol{q} \ \ , \ \ \dot{\boldsymbol{q}} = \Delta\dot{\boldsymbol{q}} \ \ , \ \ \ddot{\boldsymbol{q}} = \Delta\ddot{\boldsymbol{q}} \tag{1}$$

where $\Delta\boldsymbol{q}$ contains joint variations around the chosen configuration. By neglecting Coriolis and centrifugal terms [4], the equations that drive the dynamic model under free vibrations are:

$$\boldsymbol{M}(\boldsymbol{q}_0)\Delta\ddot{\boldsymbol{q}} + \boldsymbol{C}_q\Delta\dot{\boldsymbol{q}} + \boldsymbol{K}_q\Delta\boldsymbol{q} = 0 \tag{2}$$

where $\boldsymbol{M}(\boldsymbol{q}_0)$ is the $n \times n$ mass matrix $\boldsymbol{q}_0$-dependent, $\boldsymbol{C}_q$ is a diagonal matrix that accounts for joint damping and $\boldsymbol{K}_q$ is a stiffness matrix that accounts for joint compliance. Gravity torques are negligible for free vibrations.

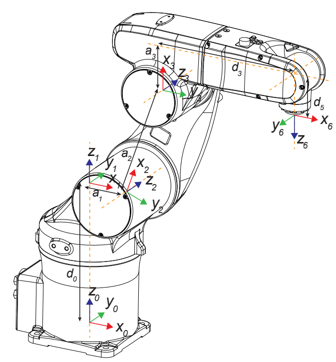

Fig. 1. The robot Adept Viper s650 reference frames.

In particular, usually $\boldsymbol{M}(\boldsymbol{q}_0)$ can be calculated from the robot inertial properties, namely link mass m_i, center of mass $\boldsymbol{G}_i$, and inertia tensor $\boldsymbol{I}_i$; the partial Jacobians $\boldsymbol{J}_{p,i}$ and $\boldsymbol{J}_{O,i}$; the rotation matrix $\boldsymbol{R}_i$ from link i to base frame:

$$\boldsymbol{M}(\boldsymbol{q}_0) = \sum_{i=1}^{5}(m_i\boldsymbol{J}_{p,i}^T\boldsymbol{J}_{p,i} + \boldsymbol{J}_{O,i}^T\boldsymbol{R}_i\boldsymbol{I}_i\boldsymbol{R}_i^T\boldsymbol{J}_{O,i}) \tag{3}$$

Due to the mechanical structure of the robot $\boldsymbol{M}(\boldsymbol{q}_0)$ is a non-diagonal matrix, which means that a vibrational mode i (dominated by joint i) includes small contributions of other joints. The following section will analyze this aspect.

III. Numerical results

The model described in the previous Section has been implemented for an Adept Viper s650 6-axis robot. While the joint stiffnesses have been retrieved from experimental analysis, the inertia properties have been calculated both from the datasheet and CAD data obtained from the vendor website.

Equation 3 shows that $\boldsymbol{M}$ highly depends on the configuration. Since $\boldsymbol{K}_q$ is configuration-independent, it results that

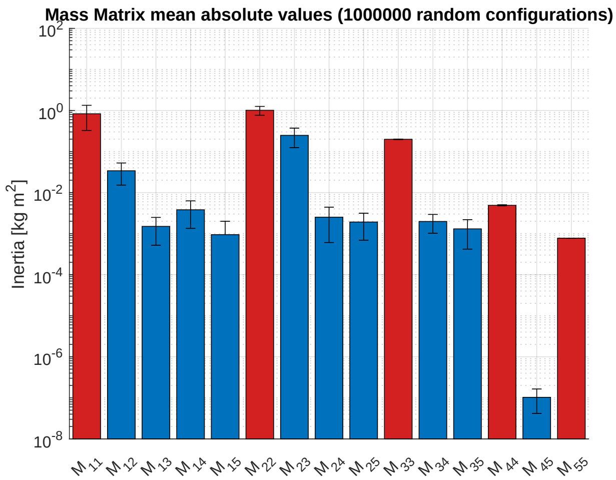

Fig. 2. Mean values and standard deviations of mass matrix terms (10^6 random configurations)

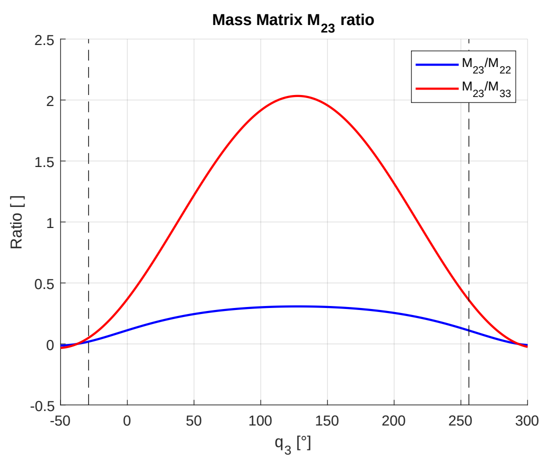

Fig. 3. Effect of joint value q_3 on inertial cross coupling between joints 2 and 3. Black dashed lines describe the mechanical limits of the joint

Mode	Conf: [0,30,30,0,90,0]°		Conf: [30,-50,160,0,90,0]°	
	Calc. f [Hz]	Exp. f [Hz]	Calc. f [Hz]	Exp. f [Hz]
1	15.4	16.3	12.9	13.3
2	16.1	17.8	15.7	14.5
3	33.0	36.2	46.1	40.2
4	57.2	58.4	57.2	59.1
5	171.9	170.5	172.3	171.8

TABLE I
VALIDATION RESULTS

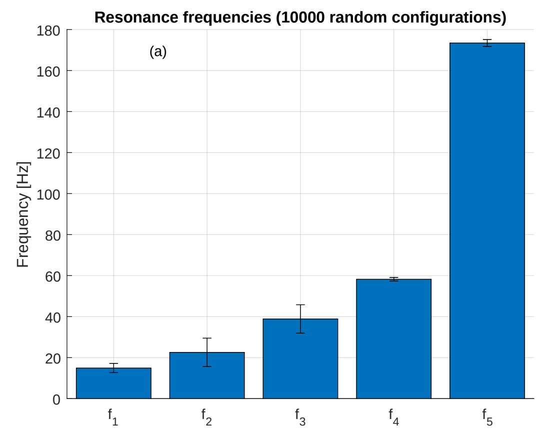

Fig. 4. Mean values and standard deviations of natural frequencies in the workspace with random configurations

also the resonance frequencies depend on the configuration. In our case $\boldsymbol{M}$ is a 5×5 matrix since joint 6 is axial-symmetric so its contribution is negligible.

Figure 2 shows how the terms of the inertia matrix $\boldsymbol{M}$ vary with the configuration $\boldsymbol{q}_0$. The red bars show the diagonal terms, whilst the blue bars show the coupling terms. It is clear how the latter terms are not negligible in the inertial analysis of an industrial robot. It is worth to notice how there is a strong coupling between joints 2 and 3 (Figure 3) since the coupling term $M_{23} = M_{32}$ is of the same order of magnitude of the diagonal terms M_{22} and M_{33}, respectively related to joint 2 and 3. This result is rather intuitive since these two joints are parallel and consecutive.

The coupling between joints 2 and 3 can be found also if the resonance frequencies are analyzed (Figure 4): whilst the resonance frequencies f_1, f_4, and f_5 show small variations, f_2 and f_3 show wider ranges. Validation tests have been performed to validate the model: Table I shows strong correlation between calculated and experimental frequencies.

IV. Conclusions

If vibrations are an important aspect of an industrial robot application, robot configurations should be taken into account in the task design. This paper shows that resonance frequencies vary with the configuration, and joints 2 and 3 (commonly parallel and consecutive in most industrial robots) are bounded by inertial coupling that affects the dynamic behaviour of the robot.

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