A Coordinate Invariant Index for Manipulability Analysis in Surgical Robotics with a Remote Center of Motion Constraint

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***Abstract*—In this work we perform a manipulability analysis in the context of a surgical task subject to a remote center of motion constraint.**

***Index Terms*—Manipulability Analysis, Surgical Robotics.**

I. Introduction

Medical robotics application is currently a growing field of robotics, aimed to improve precision, enhance dexterity, reduce invasiveness of operation and overall time of intervention with a following reduction of the recovery time for the patient. In these applications, multiple tasks are assigned to the robot to ensure a safety and adaptable behaviour of it, e.g. joint limits and obstacle avoidance, or manipulability maximization. One of the big challenges in robotics assisted surgery is the constrained manipulation of tissue through a pivot point, referred to as Remote Center of Motion (RCM). The significance of employing a robot is that it compensates for the reduced number of DOFs that result from the RCM constraint, enhancing dexterity. We define a coordinate invariant index, aimed to give an objective and consistent measure of the robot manipulability.

II. METHODS

In the context of robotics-assisted minimally invasive surgery the robot tool is inserted into the patient body through an incision point, P_{trocar}. The Trocar point is defined as a fix point in the world frame through which the shaft of the tool has to pass, and constitute the remote center of motion for the manipulator. In section II-A the constrained kinematic as in [1] is presented, followed by the proposed coordinate invariant index in Section II-B and the obtained results in Section III.

A. Kinematic Constraint at a RCM

We will denote with $P_{RCM} \in \mathbb{R}^3$ the RCM point that must coincide with the trocar point $P_{trocar} \in \mathbb{R}^3$. The RCM is assumed to belong to a shaft attached at the end effector of the manipulator, and can be located anywhere on the tool. Following the formulation proposed in [1], the position of the RCM over time is given by:

$$\mathbf{p}_{RCM} = \mathbf{p}_i + \lambda(\mathbf{p}_{i+1} - \mathbf{p}_i) \qquad 0 \leq \lambda \leq 1 \tag{1}$$

where $\mathbf{p}_i$ and $\mathbf{p}_{i+1}$ denote the boundaries of the shaft. The dependencies of the points coordinates from joint variables and time are omitted for brevity.

Differentiating (1), and exploiting the differential mapping between the joint space and the operational space, we obtain:

$$\dot{\mathbf{p}}_{RCM} = \mathbf{J}_{RCM}(\mathbf{q}, \lambda) \begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\lambda} \end{pmatrix} \tag{2}$$

where $\mathbf{J}_{RCM}$ is the Jacobiano of the RCM, given by:

$$\mathbf{J}_{RCM} = \begin{pmatrix} \mathbf{J}_i + \lambda(\mathbf{J}_{i+1} - \mathbf{J}_i) & \mathbf{p}_{i+1} - \mathbf{p}_i \end{pmatrix} \tag{3}$$

To satisfy the RCM constraint, it has to be $P_{RCM}(t) \equiv P_{trocar}(t)$, therefore $\dot{\mathbf{p}}_{RCM} = \mathbf{0}$.

Indicating with $\mathbf{t} = \mathbf{f}(\mathbf{q})$ a generic desired task, and considering the differential kinematic between task and joints velocities, it is possible to derive the differential kinematic of the extended task which includes the above mentioned RCM constraint.

$$\dot{\mathbf{t}}_{EXT} = \begin{pmatrix} \dot{\mathbf{t}} \\ \mathbf{0}_{3\times1} \end{pmatrix} = \begin{pmatrix} \mathbf{J}_t & \mathbf{0}_{n_t\times1} \\ \multicolumn{2}{c}{\mathbf{J}_{RCM}} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\lambda} \end{pmatrix} = \mathbf{J}_{EXT} \begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\lambda} \end{pmatrix} \tag{4}$$

where n_t is the dimension of the task space.

To guarantee exponential decoupled convergence of the extended task to a desired value, we employed the following kinematic control:

$$\begin{pmatrix} \dot{\mathbf{q}} \\ \dot{\lambda} \end{pmatrix} = \mathbf{J}^{\dagger}_{EXT} \begin{pmatrix} \mathbf{K}_t & \mathbf{0}_{nt\times3} \\ \mathbf{0}_{3\times nt} & \mathbf{K}_{RCM} \end{pmatrix} \mathbf{e}_t \tag{5}$$

where $\mathbf{e}_t$ is the vector containing the error of the task and of the rcm, i.e. $\mathbf{e}_t = \begin{pmatrix} \mathbf{t}_d - \mathbf{t} & \mathbf{p}_{trocar} - \mathbf{p}_{RCM} \end{pmatrix}^T$.

In (5) additional tasks could be considered and projected into the null-space of the extended jacobian $\mathbf{J}_{EXT}$.

B. Coordinate Invariant Manipulability Index

As mentioned in the introduction, it is crucial to monitor the proximity of a singularity and check the feasibility of a stack of tasks. Most of the kinematic performance measures proposed in the last decades are related to the concept of manipulability, dexterity and isotropy. In a singular configuration the considered performance measure is zero, and it increases as soon as the robot moves out of the singularity.

Manipulability measures are related to the concept of manipulability ellipsoids, and its volume was assumed to be a measure of uniformity of the mapping between joint and task spaces. The ellipsoids are used to analyse pure kinematic feasibility to arbitrarily generate end-effector velocity or force in a certain joint configuration. It is not often remarked, but manipulability ellipsoids are dependent from the joint coordinate choice [2], i.e. arbitrarily manipulability could be inferred for the same configuration depending on the chosen coordinates in the joint space. Therefore, it is of crucial importance the choice of the kernel of the quadratic form which describes the ellipsoids' equation in order to have an objective measure.

Considering the mass matrix $\mathbf{M}(\mathbf{q})$ of the manipulator as a metric tensor to redefine the sphere related to the joint velocities, we have the same ellipsoid no matter the parametrization chosen for $\dot{\mathbf{q}}$ [2]. Following this reasoning, it is possible to define velocity and force ellipsoids in the Cartesian space, as in (6). The kernel in these equations is given by the inertia in the Cartesian space ($\mathbf{\Lambda} = (\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)^{-1}$) and its inverse. This matrix is the induced metric in the Cartesian space, obtained by using $\mathbf{M}$ as a metric in the Joint configuration space.

$$\begin{aligned} \dot{\mathbf{x}}^T(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)^{-1}\dot{\mathbf{x}} = 1, \\ \mathbf{f}^T(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T)\mathbf{f} = 1. \end{aligned} \quad (6)$$

The velocity and force ellipsoids in (6) incorporate a strong physical meaning: their principal axis describe the capability of the robot to produce end-effector velocities/forces in certain directions taking into account not only the kinematic structure of the robot, but also the dynamic constraints intrinsically expressed by the inertia matrix $\mathbf{\Lambda}$; furthermore, they are independent of the joint parametrization, which comes as consequence of choosing a proper metrics in the Joint Configuration Space [2].

The manipulability measure related to the classical ellipsoid expression is given by the square root of the determinant of the ellipsoid's kernel, i.e. $w = \sqrt{det(\mathbf{J}\mathbf{J}^T)}$. Considering the above mentioned metric, we consider the inertia matrix as a coordinate invariant measure of manipulability: $w = \sqrt{det(\mathbf{\Lambda}^{-1})}$.

III. RESULTS

Here we want to compare our approach with the one proposed in [3], during the execution of a trajectory which could be compared to the first arch of circle made during a suture, while satisfying the RCM constraint. In [3] a modified variant of the measure of isotropy as the Frobenius condition number of the Jacobian matrix is proposed. They argue that due to the slow motion of a surgical task, they can decompose it in a *reach*, only translational, or *orient* motion. In this way the resulting manipulability index is unit-invariant.

As can be seen from Figure 1, the two indexes have a different behaviour throughout the trajectory. In Figure 1a and 1c the trend of the manipulability index computed as in [3] are depicted, more in detail in 1c it is depicted the trend of the measure considering only the translational Jacobian. In Figure 1b and 1d our proposed index is reported. The two indexes not only have different values, but also have opposite trends when the whole Jacobian is considered, and in both cases, if the measure is considered only for the reach motion, we have a smaller value of the manipulability.

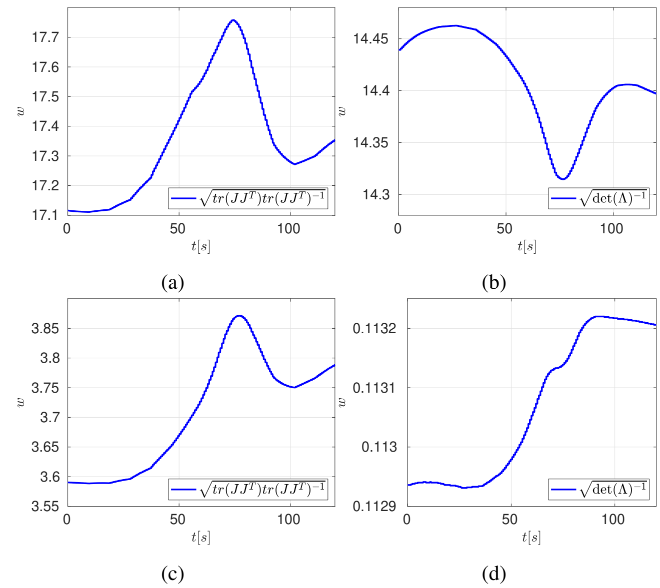

Fig. 1: Comparison of the manipulability indexes: in 1a and 1c manipulability computed considering the Frobenious norm, 1c considering only the translational jacobian; in 1b and 1d the manipulability computed with our approach, 1d - as in 1c - considering only the translational jacobian.

IV. CONCLUSIONS AND DISCUSSION

A coordinate invariant manipulability index has been proposed, which could be useful for the on-line detection of algorithmic singularities. Further research direction would be to investigate an appropriate reactive control schemes to manage the conflict situations. We aim to evaluate the proposed manipulability index in a human-robot interaction framework. In addition, we want to consider also other crucial tasks such as distance from joint limits, obstacle avoidance and manipulability maximization. In this framework, we aim to study the feasibility of the stack with the imposed RCM constraint.

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