The Reachable Region: a Fast Kinematic Feasibility Criterion for Legged Locomotion

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***Abstract*—Synthesizing kinematically feasible trajectories for legged robots becomes more and more challenging with the increase in the complexity of the obstacles to be traversed. In this abstract we propose an algorithm for the efficient assessment of the kinematic feasibility of center of mass trajectories for legged robots. This allows us to evaluate and optimize motion plans for quadruped robots over unstructured terrains online in a receding-horizon fashion.**

***Index Terms*—Legged Locomotion, Trajectory Optimization**

I. Introduction

Developing feasible body trajectories for legged systems on arbitrary terrains is a challenging task. Given some predefined contact points for the feet, the body trajectory (position and orientation), designed to move the robot, must satisfy crucial constraints to maintain *balance*, and to not violate physical hardware limits. In our previous work[1], we have introduced the Feasible Region which represents an intuitive yet powerful tool to plan feasible Center of Mass (CoM) trajectories.

The approach used in that work is to employ incremental projection algorithms [2] to compute a $2D$ region which is a set of CoM locations where the requested feasibility conditions are met, and to use it for motion planning. Furthermore, the distance between the projection of the CoM point on the region from its boundaries can be used to evaluate the robustness of the robot pose in static and dynamic gaits.

The Feasible Region, however, only takes into account *friction* constraints on the contact forces and motors' *actuation* limits but it does not ensure any guarantee on *kinematic* limits.

This can be problematic when the robot negotiates high obstacles or is forced to walk in confined environments. In such situations, inconvenient adjustments in *height* and *orientation* may push the robot to violate its kinematic limits. In this respect, the seminal work of Carpentier *et al.* [3] focused on incorporating the kinematic constraints via learning proxy constraints. However these constraints are only approximated, thus the guarantees that we mention for feasibility are only valid for a simplified representation of the robot.

In this abstract we introduce the *Reachable Region*, defined as the $2D$ non-convex set of all the (x, y) positions that can be attained by the CoM without violating the joint-position limits and without reaching leg singularities.

The Reachable Region can be intersected with the feasible region [1] thus enabling the efficient generation of motion plans for legged locomotion over rough terrains that are verified again friction, actuation and kinematic limits. The computational efficiency of our formulation allows us to plan such motions in a receding-horizon fashion with a one-step prediction horizon.

A. Reachable Region

Kinematic limits are a common limiting factor as, even when the actuation capabilities are sufficiently large, reachable workspace of the robot CoM could still be very limited.

Singularities represent a further issue related to the loss of mobility due to, for example, the complete extension or retraction of one of the legs (*e.g.,* a humanoid climbing stairs).

Given the kinematic nature of these problems, we can utilize the forward kinematic relations f_i to map the kinematic constraints of the robot (defined in the joint space) to the task-space (the Cartesian space where the CoM is defined).

For a given foot position ${}^{\mathcal{W}}\mathbf{x}_{f_i} \in \mathbb{R}^3$ and trunk orientation ${}^{\mathcal{W}}\mathbf{R}_B \in SO(3)$, we can indeed find a relationship between the joint-space angles of each leg and the CoM task-space position:

$$\mathbf{c}(\mathbf{q}_i) = {}^{\mathcal{W}}\mathbf{x}_{f_i} - {}^{\mathcal{W}}\mathbf{R}_{\mathcal{B}}(f_i(\mathbf{q}_i) - {}^{\mathcal{B}}\mathbf{c}) \tag{1}$$

where ${}^{\mathcal{B}}\mathbf{c} \in \mathbb{R}^3$ is the CoM position in the base frame. Assuming that the feet in contact do not move, for a CoM position $\mathbf{c}$ to be reachable, there must exist a set of joint angles $\mathbf{q}_i \in \mathbb{R}^{n_l}$ (n_l being the number of degrees of freedom of each limb), satisfying (1), for each leg i in contact with the ground such that:

a) $\underline{\mathbf{q}}_i \leq \mathbf{q}_i \leq \bar{\mathbf{q}}_i$
b) $J_i(\mathbf{q}_i) = [\partial f_i(\mathbf{q}_i)/\partial \mathbf{q}_i]$ is full rank

where $\underline{\mathbf{q}}_i$ and $\bar{\mathbf{q}}_i$ are the minimum and maximum joint angle limits, respectively and $\leq$ is an element-wise relational operator.

We can therefore utilize (1) (we drop the explicit dependence on ${}^{\mathcal{W}}\mathbf{x}_{f_i}$ and ${}^{\mathcal{W}}\mathbf{R}_B$ that are input parameters, to lighten the notation), along with conditions (a) and (b), to define the *reachable region* as:

$$\mathcal{Y}_r = \left\{ \mathbf{c}_{xy} \in \mathbb{R}^2 | \quad \exists \mathbf{q}_i \in \mathbb{R}^{n_l} \text{ s.t.} \quad (\mathbf{c}_{xy}, \mathbf{q}_i) \in \mathcal{Q} \right\} \tag{2}$$

where:

$$\mathcal{Q} = \Big\{ \mathbf{q}_i \in \mathbb{R}^{n_l}, \mathbf{c}_{xy} \in \mathbb{R}^2 | \text{ s.t. } \quad \mathbf{c}_{xy} = \mathbf{P}_{xy}\mathbf{F}_i(\boldsymbol{q}_i), \\ \underline{\boldsymbol{q}}_i \leq \boldsymbol{q}_i \leq \bar{\boldsymbol{q}}_i, \quad \text{row-rank}(J_i(\mathbf{q}_i)) = n_l \quad \forall i = 1, ..., n_c \Big\} \tag{3}$$

where only the legs in contact are considered. It is important to note that such set can result from the intersection of pairs of concentric circles [4] which leads to the reachable region being, in general, a non-convex set. We therefore employ a numerical approach that provides an approximation of the region, that is designed to be efficient for any generic platform.

Inspired by ray-casting algorithms, a discretized search is done iteratively in ordered directions along polar coordinates (ρ, θ) starting from the current CoM projection. This generates a 2D polygon whose vertices are ordered and belong to the boundary of the reachable region, therefore representing a *polygonal* approximation of the said region. For the sake of simplicity we refer to the reachable region $\mathcal{Y}_r$ as its polygonal approximation.

Each ray along some direction $\mathbf{a}_i$ finds the farthest point $\boldsymbol{\nu}^*_{xy}$ that yet belongs to the region. By construction, this point belongs to the boundary of the region and the problem of computing it can be stated, utilizing the inverse kinematics, as:

$$\max_{\boldsymbol{\nu}_{xy}} \ \mathbf{a}_i^T \boldsymbol{\nu}_{xy} \tag{4}$$

s.t. $\forall i = 1, ..., n_c$:

$$\mathbf{q}_i = \bar{\mathbf{F}}_i(\boldsymbol{\nu}_{xy}) \tag{5}$$

$$\underline{\mathbf{q}}_i < \mathbf{q}_i < \bar{\mathbf{q}}_i \tag{6}$$

$$\sigma_{\min} \left\{ J\left(\mathbf{q}^k\right) \right\} > \sigma_0 \tag{7}$$

The relation (5) represents the kinematic constraint in (1) reformulated in terms of the inverse kinematics. $\bar{\mathbf{F}}_i$, therefore, is defined as:

$$\bar{\mathbf{F}}_i(\boldsymbol{\nu}_{xy}) = f_i^{-1}[{}^{\mathcal{B}}\mathbf{R}_{\mathcal{W}}({}^{\mathcal{W}}\mathbf{x}_{f_i} - \mathbf{P}_{xy}^T\boldsymbol{\nu}_{xy} - \mathbf{P}_z^T c_z) + {}^{\mathcal{B}}\mathbf{c}] \tag{8}$$

where f_i^{-1} refers to the inverse kinematics mapping and P_{xy}, P_z are selection matrices. It is important to note from (8) that for specific feet positions, the location of each $\boldsymbol{\nu}^*_{xy}$ (and accordingly the resulting region) is influenced by the height c_z and by the orientation ${}^{\mathcal{W}}\mathbf{R}_B$ of the robot. A simple check for the presence of a singularity is done in (7), where $\sigma_{\min}$ is the smallest singular value and σ_0 is a small value of choice.

Unlike the feasible region [1], due to the non-linearity of constraints (5) and (7) the problem cannot be cast as a linear program (LP) and we here employ a ray-casting approach to find the solution. We first perform an evenly distributed search along the selected direction $\mathbf{a}_i$, with steps $\Delta\rho$, to find both the last point inside the region and the first point outside. These correspondingly generate the interval $[\rho - \Delta\rho, \rho]$ where $\boldsymbol{\nu}^*_{xy}$ lies in. A fast bisection search is then executed on this interval to find $\boldsymbol{\nu}^*_{xy}$. Each vertex $\boldsymbol{\nu}^*_{xy}$ is added to the vertex description $\mathcal{Y}_r$ such that the (non-convex) hull of the ordered set of vertices becomes an approximation of the real reachable region. The algorithm stops when a step smaller than $\Delta\rho_{min}/2$ set by the user, is reached.

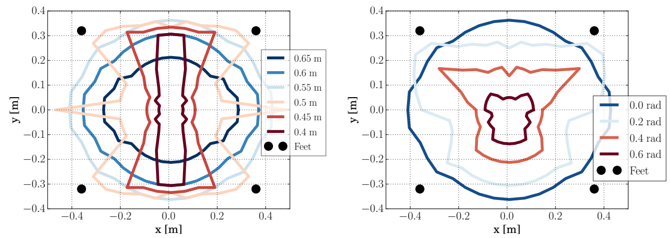

Fig. 1: Evaluations of the reachable region at different base heights (left) and different values of the base roll angle (right).

It is important also to consider the effect of the robot height c_z and orientations ${}^{\mathcal{W}}\mathbf{R}_{\mathcal{B}}$ on the reachable region. Figure 1 shows different evaluations of the reachable region where we can see that the size, positioning, shape, and convexity of the reachable region can differ greatly for different values of c_z and ${}^{\mathcal{W}}\mathbf{R}_{\mathcal{B}}$. Such insight is greatly useful in situations where planning needs to be performed in cluttered environments. We show the effectiveness of a planning strategy CoM based on the reachable region in experiments with the real robot platform Hydraulically actuated Quadruped (HyQ), where we have the robot walk at 0.03 m/s with a significantly low height of 43 cm compared to a default robot height of 58 cm[1].

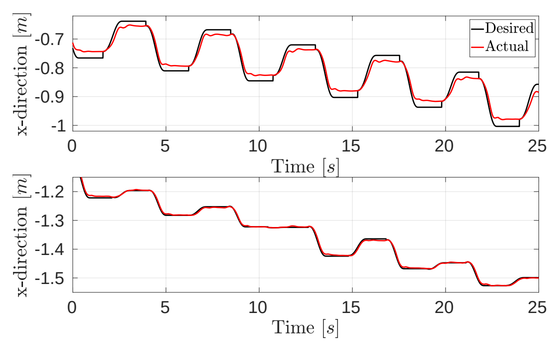

Fig. 2: Experimental results with HyQ walking at low height of 43cm. The plots in Fig. 2 shows, in the case of a standard heuristic strategy [5], multiple violations of the kinematic limits with the consequent deterioration of the tracking (upper plot) while the one based on the kinematic region has no violation at all (lower plot).

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