# Decoupled dynamic model of an aerial manipulator

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***Abstract*—In aerial manipulators, precision may be jeopardized by the forces and torques that the manipulator transfers to the UAV. In this research, a simplified decoupled dynamic model of a UAV and a 1-DOF manipulator is developed and validated against a coupled dynamic model. Then, a decoupled model is proposed for a 3-DOFs manipulator. With both the decoupled and coupled model, a lateral displacement of the system is generated during the manipulation in hovering flight, which needs to be compensated.**

***Keywords—UAV, aerial manipulation, robot***

## I. INTRODUCTION

Aerial manipulation is a new and emerging field of research [1-3]. Potential application scenarios, are inspection and maintenance, search and rescue, structure assembly, and logistics. Due to the coupled kinematics and dynamics of UAV and manipulator, the precise positioning of the end-effector of the manipulator for grasping and manipulation is very challenging.

In order to reduce the destabilizing effect and improve the robotic grasping precision, a manipulator design that minimizes the variation of Center of Gravity (COG) of the robotic arm during its motion using a static balancing approach is proposed in [4]. Other authors proposed to minimize the arm COG displacement [5] or the attitude variation [6] using redundant arms and a tailored control. Nevertheless, in all of these works, the experimental results still show significant COG variations and attitude destabilization during the manipulator motion.

In this research, a simplified decoupled dynamic model [7] of a UAV and a 1-DOF manipulator is developed and validated against a coupled model. Then, a decoupled dynamic model [7] is proposed for a 3-DOFs manipulator, which is testes by simulating a real pick and place operation.

## II. SYSTEM DESCRIPTION

The aerial manipulator considered in this study is composed of an heavy payload octocopter UAV (DJI S1000, Fig. 1) and a planar 3-DOFs robot manipulator (Fig. 2). The manipulator weights 1.315 kg and is 0.52 m long in extended configuration. A 1-DOF manipulator (mass of 1 kg and 0.13 m long) is used for comparison purposes in Section IV.

## III. UAV DYNAMIC MODEL

The dynamics of the UAV is described by Newton's and Euler's equations, which, assuming hovering flight, axes of propellers parallel to each other, and lift and drag forces proportional to the square of the angular velocity of propellers, become:

Fig. 1. UAV considered in this study

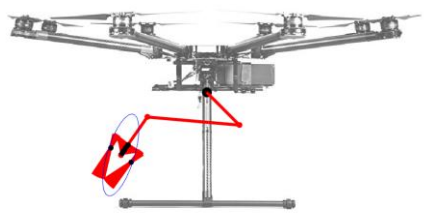

Fig. 2. Schematic drawing of UAV and 3-DOFs manipulator

$$
\begin{aligned}
m_{UAV}\,\ddot{x} &= \theta\, U_1 \\
m_{UAV}\,\ddot{y} &= -\phi\, U_1 \\
m_{UAV}\,\ddot{z} &= -m_{UAV}\, g + U_1 \\
I_x\,\ddot{\phi} &= \dot{\theta}\,\dot{\psi}\,(I_y - I_z) - I_{zr}\,\dot{\theta}\,\Omega + U_2 \\
I_y\,\ddot{\theta} &= \dot{\phi}\,\dot{\psi}\,(I_z - I_x) + I_{zr}\,\dot{\phi}\,\Omega - U_3 \\
I_z\,\ddot{\psi} &= \dot{\phi}\,\dot{\theta}\,(I_x - I_y) + U_4
\end{aligned}
\tag{1}
$$

with

$$\Omega = (\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4 + \Omega_5 - \Omega_6 + \Omega_7 - \Omega_8) \tag{2}$$

The position of the UAV is defined by its $x, y, z$ coordinates in the fixed reference frame, and its attitude by the roll, pitch, and yaw angles $(\phi, \theta, \psi)$; $m_{UAV}$ is the UAV mass, $I_x, I_y, I_z$ are the moment of inertia of the UAV around its Center of Mass (CM), and $\Omega_i$ is the angular velocity of the $i^{th}$ propeller. The term $U_1$ is the global thrust force. In hovering flight it is almost aligned to the vertical direction. The terms $U_2, U_3, U_4$ are the torques about the body axes generated by the propellers. The term $I_{zr}$ is the barycentric moment of inertia of propellers around their $z$ axis.

A control system is needed to set the inputs $U_1, U_2, U_3, U_4$ in order to obtain the desired vertical position and attitude of the UAV. In this research, small oscillations about the hovering configuration are considered, and a set of independent PID controls is used. The UAV dynamics (1) and related control have been implemented in Simulink.

## IV. UAV WITH 1-DOF MANIPULATOR

The coupled model of UAV and 1-DOF manipulator (Fig. 3 (left)) has 4 DOFs, which are associated to the coordinates of the CM of the UAV ($y$ and $z$), roll angle $\phi$, and rotation $\theta$ of the robot arm with respect to the UAV.

The system dynamics has been modeled using Lagrange's equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{3}$$

In (3), $L$ is the Lagrange's function, $q_i$ are the generalized coordinates ($y$, $z$, $\phi$, and $\theta$), and $Q_i$ the generalized forces. The generalized forces along the $y$ and $z$ coordinates are the components of the global thrust force $U_1$. The generalized force along $\phi$ is $U_2$, whereas the torque exerted by the motor of the manipulator joint is the generalized force $Q_\theta$. Inputs $U_1$ and $U_2$ are controlled by the PID controllers mentioned in Section III.

The effect of a prescribed motion of the manipulator on the position and attitude of the UAV is studied. The simulated motion of the manipulator is a rotation from the vertical position (with the end-effector pointing downwards) to a maximum angular position ($\theta$ = 65 deg), and back to the vertical position. A triangular velocity profile is prescribed with maximum velocity equal to 0.45 rad/s and duration 10 s.

The dynamic equations have been implemented and solved in Simulink, and Fig. 4 (blue lines) shows the effect of manipulator motion on the UAV. The motion of the manipulator joint causes some oscillations of the roll angle $\phi$, which are damped by the PID control. A lateral displacement of the UAV arises, which causes a final position error of about 0.06 m. This is due to the inclination of the thrust force that generates a lateral force component, which is not compensated by the control system.

On the other hand, the proposed decoupled model synergistically uses Working Model 2D (WM) and Simulink. First, the force and torque reactions exerted by the manipulator on the UAV are computed using WM. Then, the calculated force and torque reactions are used as an input to the Simulink UAV model in order to estimate UAV dynamics during the manipulation. In the WM analysis (see Fig. 3 (right) for a 3 DOFs manipulator), the UAV control is simplified assuming that the manipulator base is free to slide along a horizontal guide (to simulate the fact that the UAV is not actuated along this direction), and that the manipulator base cannot rotate (roll angle $\phi$ always equal to zero, to simulate an ideal attitude control with zero error). UAV dynamics is simulated by means of the model of section III.

The simulation results of the decoupled model (red dashed lines) are compared to those of the coupled model

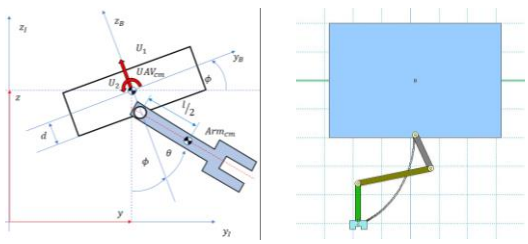

Fig. 3. Coupled model with 1-DOF arm (left), and WM model with 3-DOFs arm (right)

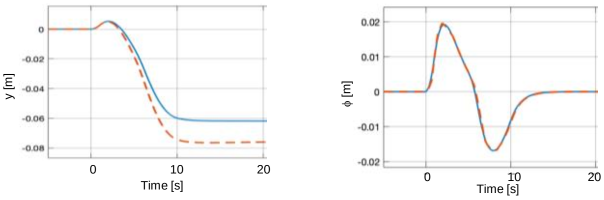


Fig. 4. Dynamic response of the coupled model (blue lines) and of the decoupled model (red dashed lines) – 1-DOF arm

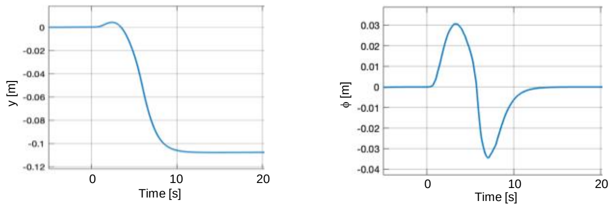


Fig. 5. Dynamic response of the decoupled model – 3-DOFs arm

(blue lines), see Fig. 4. Negligible errors are present in the variables $\phi$ and $z$, and the presence of a $y$ displacement of the system is predicted by both models, even if an approx. 10% error is present when the decoupled model is used.

Therefore, the simulation results show that the decoupled model gives a reasonable approximation of the real system dynamics.

## V. UAV WITH 3-DOFS MANIPULATOR

A pick and place operation is dealt with, in which the 3-DOFs manipulator moves from the stowed configuration to reach an object located below the UAV (see Fig. 3 (right)). The simulation results using the decoupled model are presented in Fig. 5 (lateral displacement and roll angle).

It can be noticed that at the end of the operation a net $y$ displacement of about 0.11 m is generated, which cannot be compensated by the UAV control during hovering flight.

## VI. CONCLUSIONS

A simplified decoupled model is proposed to simulate the dynamics of aerial manipulators. After validation against a coupled model for a 1-DOF manipulator, the decoupled model is used to simulate the dynamics of a UAV with a 3-DOFs planar manipulator in a pick and place operation. With both the decoupled and coupled model, an undesired horizontal displacement of the UAV arises, which needs to be compensated to enhance the manipulator precision.

## REFERENCES

[1] H. Bonyan Khamseh et al., "Aerial manipulation-A literature survey," Robotics and Autonomous Systems, vol. 107, pp. 221–235, 2018.

[2] X. Ding et al., "A review of aerial manipulation of small-scale rotorcraft unmanned robotic systems," Chinese Journal of Aeronautics, vol. 32 (1), pp. 200–214, 2019.

[3] F. Ruggiero et al., "Aerial manipulation: A literature review," IEEE Robotics and Automation Letters, vol. 3 (3), pp. 1957–1964, 2018.

[4] C. D. Bellicoso et al., "Design, modeling and control of a 5-DoF light-weight robot arm for aerial manipulation," 23rd Mediterranean Conference on Control and Automation, pp. 853–858, 2015.

[5] V. Lippiello et al., "Hybrid Visual Servoing With Hierarchical Task Composition for Aerial Manipulation," IEEE Robotics and Automation Letters, vol. 1 (1), pp. 259-266, 2016.

[6] G. Heredia et al., "Control of a multirotor outdoor aerial manipulator," IEEE IROS, pp. 3417-3422, 2014.

[7] S. Cocuzza, E. Rossetto, A. Doria, "Dynamic interaction between robot and UAV in aerial manipulation," 19th International Conference on Mechatronics, Mechatronika 2020, Publisher: IEEE, 2020.