

# Validation of a Novel Inverse Kinematics for Upper-Limb Rehabilitation Exoskeletons

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**Abstract**—In this work, we present and validate a novel inverse kinematics method for rehabilitation upper-limb exoskeletons that require joint coordination constraints. Starting from the conventional differential kinematics algorithm based on the inversion of the Jacobian matrix, we describe and test the improved algorithm based on the Projected-Gradient method, which takes into account inter-joint coordination constraints. The Harmony exoskeleton is used as a platform to demonstrate the method. In detail, we address the joint constraints needed to match anatomical shoulder movement and results show good performances of the proposed algorithm.

**Index Terms**—Upper-limb exoskeleton, inverse kinematics, kinematic redundancy, rehabilitation, inter-joint coordination.

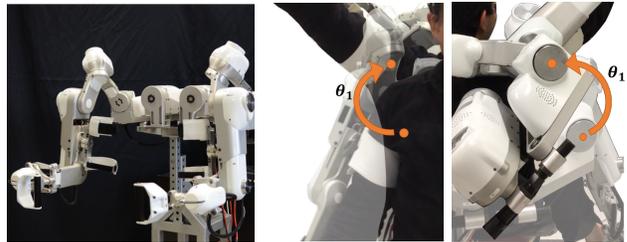
## I. INTRODUCTION

Humans can perform a variety of complex motions in both joint-space and task-space. The ability to switch between these two planning strategies allows performing complex functional goals [1]. Robots, and in particular robotic arms, have been invented to perform similar functional tasks, but they don't have the intrinsic ability to switch between joint-space and task-space. The inverse kinematics problem helps to bridge the gap between joint control and end-effector motion, however, conventional inverse kinematics approaches are still unable to solve complex constraints in the task space and joint space simultaneously [2]. Regarding upper-limb rehabilitation exoskeletons, it is even more important to provide good arm coordination than precisely controlling the end-effector position. Indeed, since inter-joint coordination limits non-coordinated compensatory movements, it is considered a key feature for post-stroke recovery. We thus propose to exploit the kinematic redundancy of the rehabilitation robot to solve for conflicting joint-space constraints. If on one hand the exoskeleton has to help the subject complete the functional rehabilitation task, on the other it has to guarantee good physiological coordination among joints. The presented method is then validated with the Harmony exoskeleton.

## II. MATERIALS AND METHODS

### A. The Harmony Exoskeleton

The Harmony exoskeleton is a bilateral powered exoskeleton designed to closely match the natural coordination of the shoulder girdle [3]. Since it has seven degrees of freedom for



(a) Harmony robot. (b) Front View. (c) Back View.

Fig. 1: (a) shows the Harmony exoskeleton, (b) and (c) show the robot compensating for the Scalupo-Humeral Rhythm.

each arm, greater than the 3-dimensional position of the end-effector, the Harmony exoskeleton is kinematically redundant (Fig. 1a). One of the key features of the robot is to maintain a high level of coordination with the wearer. The goal is to provide inter-joint assistance to gently draw the shoulder girdle to its normal trajectory corresponding to the humeral motion, allowing the users to move their arm more naturally with a wider range of motion and with no impingement on the shoulder. This nonlinear relationship, also known as the Scapulohumeral Rhythm (SHR), is defined as [4]:

$$\theta_1 = 0.0036\beta_h^2 + 0.085\beta_h \quad (1)$$

where,  $\theta_1$  is the angle of rotation for the elevation of the shoulder and  $\beta_h$  is the humeral elevation angle that can be calculated from the translation of the shoulder girdle mechanism.

This kinematic coupling is not compatible with conventional inverse kinematics techniques, and thus cannot be used for task-space motion control.

### B. Proposed Inverse Kinematics Method

In literature, researchers traditionally address the inverse kinematics problem by means of the pseudo-inverse of the Jacobian matrix. Fortunately, highly redundant robots, such as Harmony, permit the existence of a subspace  $\mathcal{N}(J)$  of joint velocities, called the Null-space, that we can explore to find a solution that satisfies additional arbitrary constraints. Baerlocher et al. [5] presented a method able to extend the inverse kinematics problem to  $n$  priority levels. Starting from the conventional inversion of the Jacobian matrix, the algorithm projects a perturbation into the Null-space that

generates motions of the robot that do not affect the position and orientation of the end-effector. We can summarize the solution of the inverse kinematics problem as follows:

$$\dot{\theta} = \mathbf{J}^\dagger \dot{x} + (\mathbf{I}_N - \mathbf{J}^\dagger \mathbf{J}) \dot{q} \quad (2)$$

where  $\dot{x}$  represents the velocities of the end-effector;  $\dot{\theta}$  are the joint velocities,  $\mathbf{J}^\dagger$  is the Moore-Penrose right pseudo-inverse and  $\dot{q}$  is the perturbation that is usually determined with a Projected-Gradient algorithm [2]. In this work, we propose a simple cost function that minimizes the distance between joint variables and target values. The objective function can be written as:

$$H(\theta) = \frac{1}{2} \sum_{i=1}^N (\theta_i - \bar{\theta}_i)^2 \quad (3)$$

where  $N$  is the number of joints,  $\theta_i$  is the joint value and  $\bar{\theta}_i$  is the desired value imposed by the joint coordination of the robot.

### C. Experimental protocol

In this work, we used the kinematic model of the Harmony exoskeleton to validate the above-mentioned algorithm. To assess the performances of the algorithm, the proposed inverse kinematics method (PG-IK) has been compared with the gold-standard solution (J-IK). The inverse kinematic problem has been solved both for circular trajectories to be followed by the end-effector (i.e. the hand) of the exoskeleton. Each circular trajectory has been performed in the frontal, sagittal and horizontal planes, at constant and variable speed. The inverse kinematics problem has been solved offline in the MATLAB environment and several evaluation metrics have been computed as described below:

1) *Number of iterations ( $I_{\#}$ ):* The average number of iterations needed to exit the convergence loop for each trajectory point. Results are presented in terms of median and interquartile values, along all the trajectories.

2) *Task-space error ( $E_{3D}$ ):* Euclidean distance between the target and the actual task-space positions.

3) *Joint-space error ( $E_{\theta_1}$ ):* Difference between desired and actual joint angles. In particular we evaluate how good the inverse kinematics method is to match joint-space equality constraints, expressed in degrees [ $^\circ$ ].

4) *Smoothness ( $S_M$ ):* Movement smoothness is computed as described in [6], where the metric is defined as the time integral of absolute value of jerk measures along the trajectory. Smoothness values are then sum among joint angles and expressed in arbitrary units.

## III. RESULTS

In this section, the performances of the two algorithms, J-IK and PG-IK, are compared for each trajectory. Since no significant changes were found among the same trajectory on different planes, we present the results obtained by aggregating the data of experiments in frontal, horizontal and sagittal planes. In Table I, performances for constant speed

TABLE I: Performance Metrics Results

Algorithm	Circular Shape - Constant Speed			
	$I_{\#}^a$	$E_{3D}^b$	$E_{\theta_1}$	$S_M^c$
J-IK	1 (0)	0.0032	8.100	0.004
PG-IK	1 (0)	0.0034	0.207	0.015
Algorithm	Circular Shape - Variable Speed			
	$I_{\#}^a$	$E_{3D}^b$	$E_{\theta_1}$	$S_M^c$
J-IK	1 (0)	0.0116	8.140	9.946
PG-IK	1 (0)	0.0107	0.254	10.58

<sup>a</sup> Median number of iterations (Interquartile Range)

<sup>b</sup> Max Euclidean error

<sup>c</sup> Cumulative Smoothness along joints, in a.u.

and variable circular trajectories are reported. It should be noted that, in all cases, very few iterations were needed to converge to a solution that satisfied the constraints and the PG-IK significantly improved the joint-space error ( $E_{\theta_1}$ ), leading to better inter-joint coordination of the robot. If the proposed algorithm was able to guarantee joint coordination, as a consequence, the addition of joint-space constraints slightly worsened the smoothness performances.

## IV. CONCLUSION

We proposed a novel method that was able to solve the inverse kinematics problem of the Harmony exoskeleton and to add inter-joint coordination constraints by exploiting the redundancy of the robot. The presented algorithm first applies the conventional differential kinematics to solve a task-space problem, then the corresponding null-space is explored to maintain additional constraints.

Results show that the Projected-Gradient (PG-IK) method was able to reduce joint-space coordination errors, while not compromising the other performances. For this reason, it can be considered a good trade-off among smoothness, computational effort and joint coordination. While the objectives and constraints presented in this work relate to joint coordination and end effector tracking, the methods presented here may be further extended. We, therefore, believe that this approach could be used for several applications, especially when robots are expected to interact with humans.

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