Non-Linear DCM Trajectory Optimization with Variable Center of Mass Height

Giulio Romualdi
Dynamic Interaction Control
Istituto Italiano di Tecnologia
Genoa, Italy
giulio.romualdi@iit.it

Stefano Dafarra
Dynamic Interaction Control
Istituto Italiano di Tecnologia
Genoa, Italy
stefano.dafarra@iit.it

Giuseppe L'Erario
Dynamic Interaction Control
Istituto Italiano di Tecnologia
Genoa, Italy
giuseppe.lerario@iit.it

Daniele Pucci
Dynamic Interaction Control
Istituto Italiano di Tecnologia
Genoa, Italy
daniele.pucci@iit.it

***Abstract*—This manuscript presents an algorithm for planning the divergent component of motion trajectory. In particular, the proposed approach takes into account a variable center of mass height while considering the zero moment point as a contact feasibility criterion. The algorithm is then framed in a non-linear optimization problem. Tests have been performed in the Gazebo simulation environment using the one-meter-tall iCub humanoid robot model. Results show the effectiveness of the approach in generating feasible trajectories while walking on steps.**

***Index Terms*—DCM Planning, Non-Linear Optimization, Bipedal Locomotion**

I. Introduction

In the last decades, a common approach for facing the humanoid robot locomotion problem is to generate an admissible Center Of Mass (CoM) trajectory approximating the motion of the robot using simplified dynamical modes. The *Linear Inverted Pendulum Model* (LIPM) is one of the simplest models to generate a CoM pattern [1]. The capture point (CP) [2] and the divergent component of motion (DCM) [3], are often used as auxiliary variables to decompose the LIPM in stable and unstable dynamics. The DCM can be considered an extension of the capture point (CP) to the three-dimensional case, maintaining the assumption of a constant CoM height. Attempts at loosening this latter assumption and extending the DCM to more complex models have also been presented [4]. This paper exploits the DCM model presented in [4] to design a planner for a feasible DCM trajectory in case of a variable CoM height. The planner is then embedded in a three-layer controller architecture [5] and tested on a simulated version of the iCub humanoid robot.

II. Humanoid robots models

In cases where the humanoid robot walks maintaining a constant height between the CoM and the stance foot, the motion of the robot can be approximated with the *Linear inverted pendulum model*:

$$\ddot{x} = \omega^2 (x - r^{zmp}), \tag{1}$$

where x is the projection of the CoM on the walking surface and r^{zmp} the zero moment point (ZMP). ω is the inverse of the pendulum time constant. i.e. $\omega = \sqrt{g/z_0}$, where g is the gravity constant.

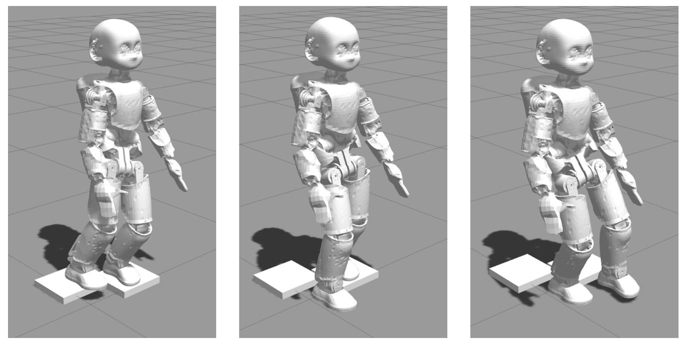

Fig. 1: iCub walks with the presented controller architecture.

The Divergent Component of Motion (DCM) is $\xi = x + \dot{x}/\omega$ [3]. Since ω is constant, the DCM time derivative is given by:

$$\dot{\xi} = \omega(\xi - r^{zmp}). \tag{2}$$

It is interesting to notice that the DCM corresponds to the unstable dynamics of the linear inverted pendulum model.

If CoM height is not constant, the Linear inverted pendulum model, and consequently (2), are no more valid approximations of the humanoid robot dynamics. In order to overcome this limitation, one may define the Divergent Component of Motion (DCM) as $\xi = x + \dot{x}/\omega(t)$. Then the DCM dynamics is [4]:

$$\dot{\xi} = \left(\omega - \frac{\dot{\omega}}{\omega}\right)(\xi - r_{vrp}) \tag{3}$$

where the virtual repellent point (VRP) r^{vrp} is defined as [4]:

$$r^{vrp} := x - \frac{\ddot{x}}{\omega^2 - \dot{\omega}}. \tag{4}$$

Let us assume there exists a $\gamma > 0$ such that $\omega - \dot{\omega}/\omega > \gamma$, it is easy to show that the equilibrium point $\xi = r^{vrp}$ is unstable. On the other hand, assuming $\omega > \rho$, the CoM asymptotically tends to the DCM.

Applying Newton's second law, (4) can be expressed as

$$r^{vrp} = x - \frac{\sum f_i - mg}{m(\omega^2 - \dot{\omega})} = r^{ecmp} + \frac{g}{\omega^2 - \dot{\omega}}, \tag{5}$$

where the enhanced Centroidal Moment Pivot (eCMP) $r^{ecmp} = x - \sum f_i / (m(\omega^2 - \dot{\omega}))$ encodes the direction and the magnitude of all the contact forces f_i acting on the CoM, given the total robot mass m and the CoM position.

III. 3D-DCM Planner

Henceforth, we will assume that a high-level footstep planner provides information regarding the desired inertial foothold poses and step timings over a finite time horizon. The generation of a feasible DCM trajectory is achieved by designing the planners as a constrained optimal control problem.

The DCM dynamics (3) is extended by considering the parameter ω as part of the state, such as:

$$\dot{\mathcal{X}} = \begin{bmatrix} \dot{\xi} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \left(\omega - \frac{u_2}{\omega}\right)(\xi - u_1) \\ u_2 \end{bmatrix} = f\left(\mathcal{X}, \mathcal{U}\right). \qquad (6)$$

where the control input u_1 and u_2 are equal to the r^{vrp} and $\dot{\omega}$. The dynamics (6) is used as prediction model, and it is discretized assuming a constant sampling time δt. The cost function is then chosen as:

$$H = \|\xi_N - \xi_N^*\|_{Q_N} + \sum_{k=0}^{N-1} \|\xi_k - \xi_k^*\|_Q + \|\dot{\omega}_k\|_{R_{\dot{\omega}}} \qquad (7a)$$

$$+ \sum_{k=0}^{N-1} \|\xi_{k+1} - \xi_k\|_{P_\xi} \qquad (7b)$$

$$+ \sum_{k=0}^{N-2} \|r_{k+1}^{vrp} - r_k^{vrp}\|_{P_{vrp}} + \|\dot{\omega}_{k+1} - \dot{\omega}_k\|_{P_{\dot{\omega}}}. \qquad (7c)$$

Where (7a) regularizes the DCM along a given reference trajectory while $\dot{\omega}$ is minimized. (7b) and (7c) smooth the generated DCM, VRP and $\dot{\omega}$ trajectories. Q, Q_n, $R_{\dot{\omega}}$, $P_{\dot{\omega}}$ P_ξ and P_{vrp} are positive definite matrices.

Eq.(3) and (4) are defined only if $\omega(t) \neq 0$ and $\omega(t)^2 - \dot{\omega}(t) \neq 0$. Assuming that $\dot{\omega}(0) = 0$, $\omega(t)^2 - \dot{\omega}(t) \neq 0$ is equivalent to $\omega(t)^2 - \dot{\omega}(t) > 0$. Furthermore if $\dot{\omega}(0) = 0$ we can assume that $\omega(0) = \sqrt{g/z_0}$, consequently $\omega(t) \neq 0$ can be simplified as $\omega(t) > 0$. It's worth noticing that the inequality relations can be easily treated by an off-the-shelf optimizer. In the case of zero desired angular momentum, the eCMP coincides with the ZMP. As a consequence the ZMP contact stability criterion can be satisfied by searching for an ECMP inside the polyhedron limited by the vertexes of the two feet. This is verified by means of a set of linear inequality constraints $A_k r_k^{vrp} \leq b_k$ where A_k and b_k are time-variant and their dimension depends on the type of support.

Since the prediction model (6) is a non-convex function, the optimal control problem is then converted into a Non-Linear programming problem and solved via off-the-shelf solvers.

IV. Results

To validate the capability of the proposed planner, we test it along with a three-layer controller architecture [5]. From top to bottom, these layers are here called: *trajectory optimization*, *simplified model control*, and *whole-body quadratic programming (QP) control*. The trajectory optimization layer consists of the planner described in Sec III. The simplified model control layer is in charge of finding a feasible linear centroidal momentum trajectory and is based on (3) and (4). Finally, the whole-body QP control layer generates robot torques that aim at stabilizing the references generated by the layers before.

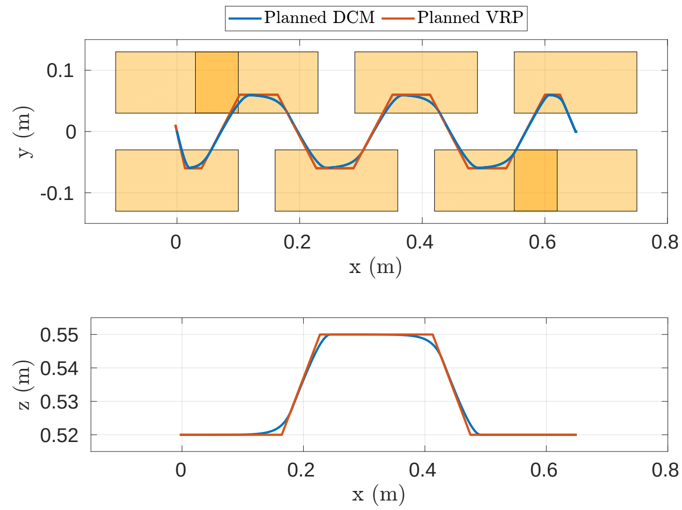

Fig. 2: Planned DCM and VRP trajectories.

The experiments are carried out on a simulated version of the iCub humanoid robot – see Fig. 1. The DCM planner takes (in average) less than $300\,\mathrm{ms}$ for evaluating a trajectory with time horizon of $4\,\mathrm{s}$ sampled at $100\,\mathrm{ms}$. The library used to solve the optimization is CasADi with IPOPT. Fig. 2 shows the desired DCM and VRP trajectories generated by the planner. In this scenario, the robot is walking straight on a set of steps. During the single support, the VRP moves from the heel to the toe of the stance foot while in double support the VRP shifts from one foot to the other.

V. Conclusions

This paper contributes towards the development of an algorithm to plan feasible DCM trajectories in case of variable CoM height. The proposed approach considers the zero moment point as a contact feasibility criterion. The algorithm is then framed into a non-linear optimization problem and solved using CasADi with IPOPT library. We finally connect the planner to a three-layer controller architecture, and test it on a simulated version of the iCub humanoid robot. As future work, we plan to validate the architecture on the real robot.

References

[1] S. Kajita, F. Kanehiro, K. Kaneko, K. Yokoi, and H. Hirukawa, "The 3D linear inverted pendulum model: a simple modeling for biped walking pattern generation," *Proc. 2001 IEEE/RSJ Int. Conf. Intell. Robot. Syst.*, no. October 2016, pp. 239–246, 2001.

[2] J. Pratt, J. Carff, S. Drakunov, and A. Goswami, "Capture point: A step toward humanoid push recovery," in *Proc. 2006 6th IEEE-RAS Int. Conf. Humanoid Robot. HUMANOIDS*, 2006, pp. 200–207.

[3] J. Englsberger, C. Ott, and A. Albu-Schäffer, "Three-Dimensional Bipedal Walking Control Based on Divergent Component of Motion," *IEEE Trans. Robot.*, vol. 31, no. 2, pp. 355–368, 2015.

[4] M. A. Hopkins, D. W. Hong, and A. Leonessa, "Humanoid locomotion on uneven terrain using the time-varying divergent component of motion," in *IEEE-RAS Int. Conf. Humanoid Robot.*, vol. 2015-Febru. IEEE, nov 2015, pp. 266–272.

[5] G. Romualdi, S. Dafarra, Y. Hu, P. Ramadoss, F. J. A. Chavez, S. Traversaro, and D. Pucci, "A benchmarking of dcm-based architectures for position, velocity and torque-controlled humanoid robots," *International Journal of Humanoid Robotics*, vol. 17, no. 01, p. 1950034, 2020.