

# Constrained Control for Optimal Collaborative Applications

Federico Benzi

*Department of Sciences and Methods for Engineering  
University of Modena and Reggio Emilia  
Reggio Emilia, Italy  
federico.benzi@unimore.it*

Cristian Secchi

*Department of Sciences and Methods for Engineering  
University of Modena and Reggio Emilia  
Reggio Emilia, Italy  
cristian.secchi@unimore.it*

**Abstract**—Shared human-robot workspaces are becoming a prominent part of the modern industrial paradigm. These collaborative environments often present highly dynamic features. It is thus fundamental that the robot maintains a behavior which is both robustly stable during the interaction with the environment and both flexible enough to ensure safety and reactivity to the varying conditions of the workspace.

In this paper, we propose a control architecture which satisfies both these conditions by combining an energy tank based variable admittance framework with control barrier functions.

**Index Terms**—Passivity, tanks, constraints, interaction

## I. INTRODUCTION

While designing a robotic application for a collaborative workspace, it is pivotal to guarantee that the robot can dynamically modify its behavior in an optimal fashion. Many researches have been conducted on this issue with various approaches such as task redundancy [1], coaching [2] and adaptive pHRI [3] for example. However, a control architecture ensuring both flexibility and stability for each possible motion of the robot is still missing.

The motion of a robot depends on the chosen interaction model and on the constraints imposed onto it by the environment, which are often time-varying in collaborative applications. By modifying the interaction parameters online, the robot can adapt its behavior to dynamically changing conditions, thus ensuring a flexible execution.

A commonly deployed strategy for controlling the interaction is admittance control [4], especially in collaborative scenarios (e.g. [3]). It enforces the robot to mimic a desired passive physical dynamics, namely the admittance dynamics. Thus, it is possible to guarantee the robustness of the interaction, since passivity is already embedded into the designed admittance dynamics (see [5]). Admittance control is however strongly limited by the choice of the interaction parameters, since these cannot be varied in real-time without causing the disruption of passivity [6], leading to unstable behaviors.

In [7], a variable admittance controller ensuring both robustness and flexibility was synthesized. Here, energy tanks [8] were leveraged for separating passivity from a fixed physical dynamics. Passivity was formulated as a constraint, using the energy in the tank to encode it into a convex optimization problem.

However, [7] presents a conservative approach, which can result in over-constraining the system, while we aim the maximizing the flexibility of the robotic application. Additionally, auxiliary dynamic constraints, such as obstacle and human avoidance, self collision and bounding the joint position, need to be satisfied for the workspace to be completely safe.

Time-varying constraints can be encoded using Control Barrier Functions (CBFs) [9], which allow to dynamically force the robot into a desired region of the state space [10]. The control input satisfying the set of constraints can be obtained as the solution of a convex optimization problem [11].

Thus, in this paper we propose a novel control architecture which leverages both energy tanks, for ensuring a robustly stable interaction, and control barrier functions, for dynamically constraining the behavior of the robot. The proposed architecture allows a collaborative robot to achieve maximal flexibility in each phase of his working cycle.

The contributions of this paper are:

- A convex optimization problem integrating the passivity constraint and the safety constraints as CBFs;
- A control architecture capable of enhancing the flexibility of the robot during both interaction and free motion;
- An experimental validation of the presented architecture onto a collaborative robot.

We hereby present the architecture and the related validation.

## II. COLLABORATIVE CONSTRAINED CONTROL ARCHITECTURE

The robot is modeled as a velocity controlled fully actuated  $n$ -DOFs manipulator:

$$\dot{x} = J(q)u \quad (1)$$

where  $x \in \mathbb{R}^m$  is the pose of the end-effector and  $q \in \mathbb{R}^n$  is the vector of joint variables.  $J(q) \in \mathbb{R}^{m \times n}$  is the Jacobian of the robot and  $u \in \mathbb{R}^n$  is the joint velocity input.

We aim at reproducing, through variable admittance control, a desired time-varying mechanical system, whose dynamics can be encompassed by the following Euler-Lagrange model:

$$M(x, t)\ddot{x} + C(x, \dot{x}, t)\dot{x} + D(x, t)\dot{x} + \frac{\partial P}{\partial x}(t) = F_e \quad (2)$$

where  $M(x, t) = M^T(x, t) > 0$  is the inertia matrix,  $C(x, \dot{x}, t)$  the Coriolis term,  $D(x, t) \geq 0$  is a damping matrix and  $P : \mathbb{R}^m \rightarrow \mathbb{R}$  is an active potential field, while  $F_e$  is the external force measured by means of a 6-DOF F/T sensor.

Since we aim at reproducing a non passive admittance dynamics, we can exploit energy tanks for enforcing a passive implementation of the desired dynamics. In fact, it is possible to guarantee the passivity of the tank by imposing:

$$T(x_t(t)) \geq \underline{\varepsilon} \quad \forall t \geq 0 \quad (3)$$

in which  $T(x_t(t))$  is the energy stored in the tank and  $\underline{\varepsilon}$  its lower bound. This constraint can then be inserted into an optimization problem to find the best passive implementation of any desired admittance dynamics, as shown in [7]

$$\begin{aligned} & \underset{\dot{x}}{\text{minimize}} \quad \|\dot{x} - \dot{x}_a\|^2 \\ & \text{subject to} \quad \int_0^t F_e^T(\tau) \dot{x}(\tau) d\tau \geq -T(x_t(0)) + \underline{\varepsilon} \end{aligned} \quad (4)$$

in which  $x_a$  is the desired admittance.

Moreover, we can encode multiple dynamic constraints representing safety tasks as Control Barrier Functions. Starting from tasks which can be formulated as the minimization of a certain cost function  $C(\sigma, t)$ , with  $\sigma$  being a measurable task variable, we can define appropriate CBFs  $h(\sigma, t) = -C(\sigma, t)$  which are non-negative only in the region of satisfaction of the task. Thus, by imposing the non-negativity of  $h$ , we can achieve the execution of the task  $\sigma$ . For each task, we can design a dedicated CBF encoding it. We can then merge the previous problem in (4) with this CBF-based framework, obtaining the following optimization problem

$$\begin{aligned} & \underset{\dot{x}, \delta}{\text{minimize}} \quad \|\dot{x} - \dot{x}_a\|^2 + l \|\delta\|^2 \\ & \text{subject to} \quad \frac{\partial h_m}{\partial t} + \frac{\partial h_m}{\partial \sigma} \frac{\partial \sigma}{\partial x} \dot{x} \\ & \quad + \alpha(h_m(\sigma, t)) \geq -\delta_m \quad m \in \{1, \dots, M\} \\ & \quad \int_0^t F_e^T(\tau) \dot{x}(\tau) d\tau \geq -T(x_t(0)) + \underline{\varepsilon} \end{aligned} \quad (5)$$

in which  $l > 0$  is a constant gain,  $M$  is the number of tasks and  $\delta = [\delta_1, \dots, \delta_M]^T$  is the vector of slack variables, which ensures the feasibility of the problem even if conflicting tasks are active at the same time. As can be easily proven, the passivity constraint automatically deactivates when the robot is not interacting with the environment, freeing the system from unnecessary constraints.

Following this procedure, we have therefore obtained a control architecture capable not only of satisfying simultaneous constraints in a flexible way but also to guarantee a robust behavior while physically interacting with a poorly known environment, thanks to the preservation of passivity. The developed framework acts as a sort of "armor", protecting the robot against unstable behaviors during the interaction, while still allowing it to implement all the encoded tasks at the best of its dexterity.

### III. EXPERIMENTS

The framework proposed in this paper has been validated on a Universal Robot 10e manipulator, equipped with an on-board 6-axis force/torque (F/T) sensor. Both the robot and the sensor run with a sampling time of 2ms.

The robot is employed to accomplish a set of tasks, including obstacle avoidance, joint control and position control of the end effector, as well as the satisfaction of the passivity constraint and the implementation of a variable admittance dynamics. Each task is encoded using a specific CBF, following the previously exposed formulation.

Furthermore, the desired admittance dynamics presents a time-varying repulsive potential  $P(x, t)$  which is centered in the current position of the obstacle. The generated force  $\frac{\partial P}{\partial x}(t)$  increases as the robot approaches the obstacle, such that the human can sense an hazardous area during the interaction.

A series of experiments have been conducted, in order to concurrently validate each component of the architecture:

- The robot moves to a desired goal while the human physically interacts with it
- The robot moves to a desired goal while avoiding a moving obstacle in the workspace
- The human tries to implement a non-passive dynamics on the robot, while the overall passivity is preserved

The experiments are more thoroughly presented in the accompanying video, alongside the related graphs.

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